

A COMPARATIVE STUDY OF THE NORMAL MODES OF VARIOUS MODERN BELLS

R. PERRIN AND T. CHARNLEY

Department of Physics, Loughborough University of Technology, Loughborough LE11 3TU, England

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Finite element calculations of the normal modes of church, carillon, hand and fire-alarm bells have been made. Some of the results are presented and comparisons made which shed new light on mode classification.

1. INTRODUCTION

Modern western bells of good quality all have a basic axial symmetry. Group theoretical arguments show [1] that this results in degenerate pairs of normal modes whose modal functions vary like $\sin(m\theta)$ and $\cos(m\theta)$ where θ is the polar angle shown in Figure 1 and m is a non-negative integer. The nodal patterns therefore contain $2m$ evenly spaced meridians with those of one doublet member lying midway between those of the other. The two members of such a doublet constitute a campanologist's partial; only for $m=0$ does a partial consist of a single mode. The absolute locations of these meridians are indeterminate until fixed by initial conditions or some perturbation. The remainder of each nodal pattern must, in the absence of accidental degeneracies unconnected with the axial symmetry, consist of n circles parallel to the rim.

This description of nodal patterns applies equally well to the radial u , tangential v and longitudinal w components of the bell's motion. However, because the acoustically important modes are all ones in which the motion has a large u component, it has become customary to classify partials in terms of the nodal pattern for u although v and w components can also be present.

Because the analytical forms of the θ -parts of the modal functions are as above, it follows that once m is specified all other information about a partial can be obtained by looking at one member of the degenerate pair in any one reference plane containing the symmetry axis. The PAFEC finite element package enables this to be exploited. Since its "axisymmetric" elements have the correct θ -variations built in, their use enables a bell to be considered as a two-dimensional structure.

By using PAFEC in this way the normal modes of a variety of modern bells have been calculated. The operation of the package requires a separate computer run for each value of m , the lowest 50 or so modes being generated for each case. It transpires that only the first few modes in the sequences with m values of 2, 3, 4, 5 and perhaps 6 are of acoustical importance in any of the bells considered in this paper. We shall therefore concentrate on this range of m values and in particular avoid detailed discussion of the unimportant $m=0$ and 1 modes which have peculiar problems associated with their classification [2].

2. THE TAYLOR CHURCH BELL

Computational results for a modern D_5 Taylor church bell, shown broken into elements in Figure 2, have previously been reported by Perrin, Charnley and DePont [2] (referred

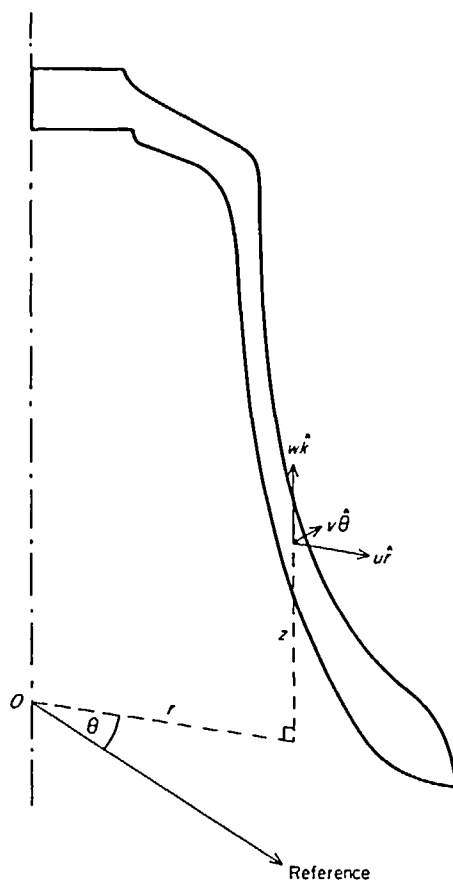


Figure 1. Cylindrical polar co-ordinates and displacement notation for any bell with the symmetry axis defining the z -direction.

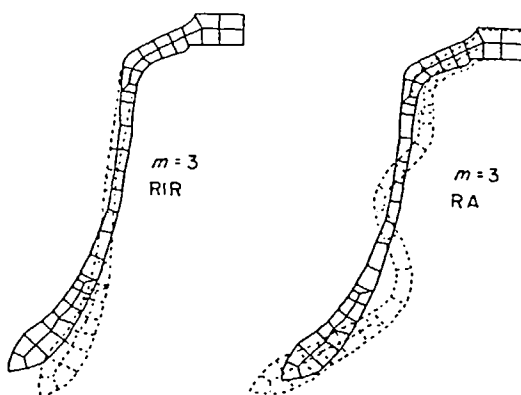


Figure 2. RIR and RA modes with $m = 3$ for a D_5 Taylor church bell.

to as PCD in this paper) for m values in the range 0 to 5 inclusive. Agreement with experiment over nodal patterns and frequencies was generally good. It was found that modes could be divided into those moving primarily in the reference plane and those moving primarily out of it. The latter correspond to extensional modes of rings in cross-sections normal to the symmetry axis and are of little acoustical importance. Their circumferential components are m times their radial ones in amplitude since these are connected by $v + \partial u / \partial \theta = 0$. Alternatively they might be regarded as corresponding to the Arnold and Warburton circumferential cylinder modes [3].

The primarily in-plane modes were found by PCD to be of three types. Two, shown in Figure 2, were regarded as being driven by the thick ring at the rim of the bell going into its inextensional radial and its axial modes respectively and were therefore designated as RIR and RA. This explained why only one mode of each type occurs for a given value of m .

The third type of primarily in-plane mode was described as "shell-driven", such modes usually having a nodal circle at or near the rim ring. For each value of m there is always a family of such modes with 1, 2, 3, ... nodal circles. A typical sequence, that for $m = 3$, is shown in Figure 3. It should be noted that as n increases through a critical value the crown of the bell begins to participate in the motion and continues to do so for higher values. These shell-driven modes, like the RIR, are inextensional in the sense that any cross-section normal to the symmetry axis contains a neutral circle whose circumference remains unchanged in length during the motion. This means that $u + \partial v / \partial \theta = 0$ and so the radial component is m times as large in amplitude as the circumferential one. Also

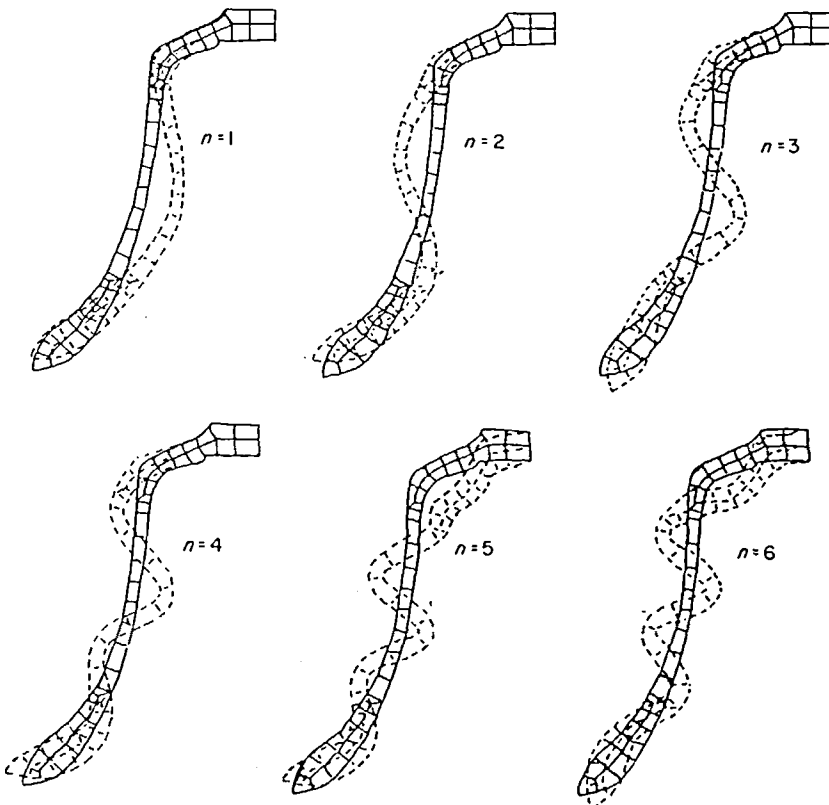


Figure 3. Shell-driven modes with $m = 3$ and $n = 1-6$ for a D_3 Taylor church bell.

the frequencies of both these types of modes are very much less than those for the extensional modes with the same m . Both of these types of mode can thus be considered as corresponding to Arnold and Warburton's radial cylinder modes. In Figure 4 are shown the forms of both inextensional and extensional modes for $0 \leq m \leq 2$ as seen in an arbitrary plane normal to the symmetry axis.

It was reported by PCD that the RIR mode with $m=2$, the Hum note, was the only one with no nodal circle and that those RIR modes with $m=3$ and 4 had one circle which was situated at the same position in the waist in each case. Calculations have now been performed with higher values of m and these clearly show that RIR modes continue to have one nodal circle *only* in this same position but that the amplitude in the region from the circle upwards decreases *very* quickly as m increases. This is shown in Figure 5 and is in sharp contrast to all the shell-driven modes, including those with $n=1$, for which the modal shapes in the reference plane remain more-or-less unchanged as m increases to even very high values; we have checked this up to $m=20$. This gives seemingly irrefutable evidence that PCD were correct to classify their RIR modes as lying outside the shell-driven family and to keep the $n=1$ cases within it, rather than interchanging them as has sometimes been suggested in the literature (see for example reference [4]). The very small amplitudes of RIR modes in the region from the waist upwards when m

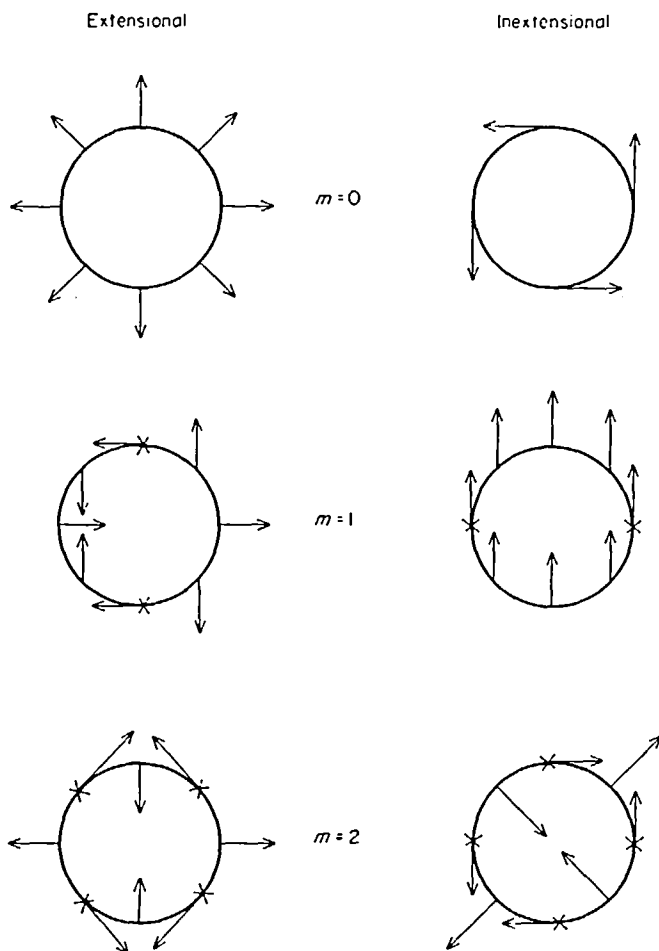


Figure 4. Motion in a plane normal to the symmetry axis for small m modes of any bell.

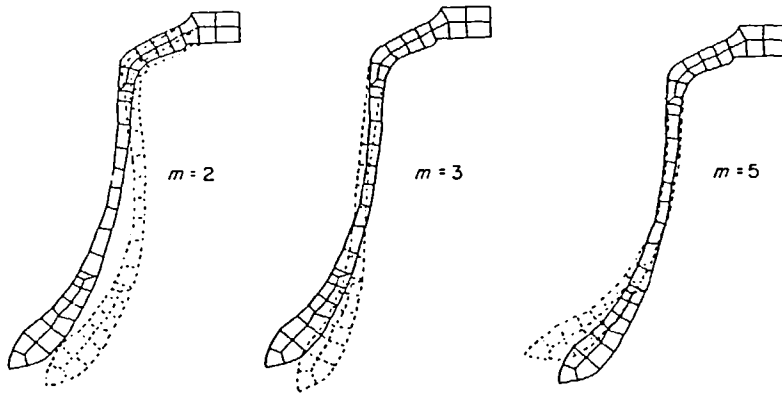


Figure 5. RIR modes with $m = 2, 3$ and 5 for a D_5 Taylor church bell.

is not small, together with the accidental degeneracies and other experimental difficulties reported by PCD, are more than adequate to explain the discrepancies between their experimental data for nodal circles and the new PAFEC calculations. The circles they reported in the shoulder region for $m > 4$ and the rapid increase in numbers of circles in the region above the waist for $m > 8$ can now clearly be seen to have been in error. This means that the suggestion of a change in regime for RIR modes [5] at about $m = 7$ is certainly wrong and the analysis of frequency data by using Chladni's Law will require a major revision in the RIR case. This will be reported elsewhere. The only regime change for RIR church bell modes is that in going from $m = 2$ with no circle to $m = 3$ with one circle in the waist.

3. THE MALMARK HANDBELL

Similar calculations by using PAFEC have been made for a C_5 Malmkär handbell and previously reported in part [6]. Agreement with experiment was again good insofar as nodal patterns and relative frequencies were concerned. The form of this bell can be seen in Figure 6.

It was found that analogues of all the church bell modes were present despite the absence of the thick rim ring. Again it is the in-plane modes which are important acoustically. The sound spectrum is now dominated just by the $m = 2$ and 3 RIR modes, which here are both without any nodal circle. They are shown in Figure 6. In the church bell the $m = 2, 3$ and 4 RIR modes and the $(m = 2, n = 1)$ and $(m = 3, n = 1)$ shell-driven modes are all musically important. It should be noted that in the $m = 4$ RIR mode for

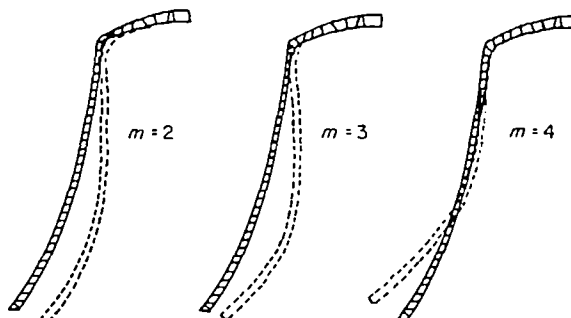


Figure 6. RIR modes with $m = 2, 3$ and 4 for a C_5 Malmkär handbell.

the handbell, shown in Figure 6, the amplitude above the single nodal circle is already very small. As m increases further the effect becomes even more marked in an analogous way to that for the church bell.

A selection of the shell-driven modes for the handbell are shown in Figure 7, from which it can be seen that they also exhibit the "crown take-off" phenomenon. However, the effect here is associated with a problem in using n to designate the family position: at the crown take-off point there is a jump of 2 in the n value. This does not occur in

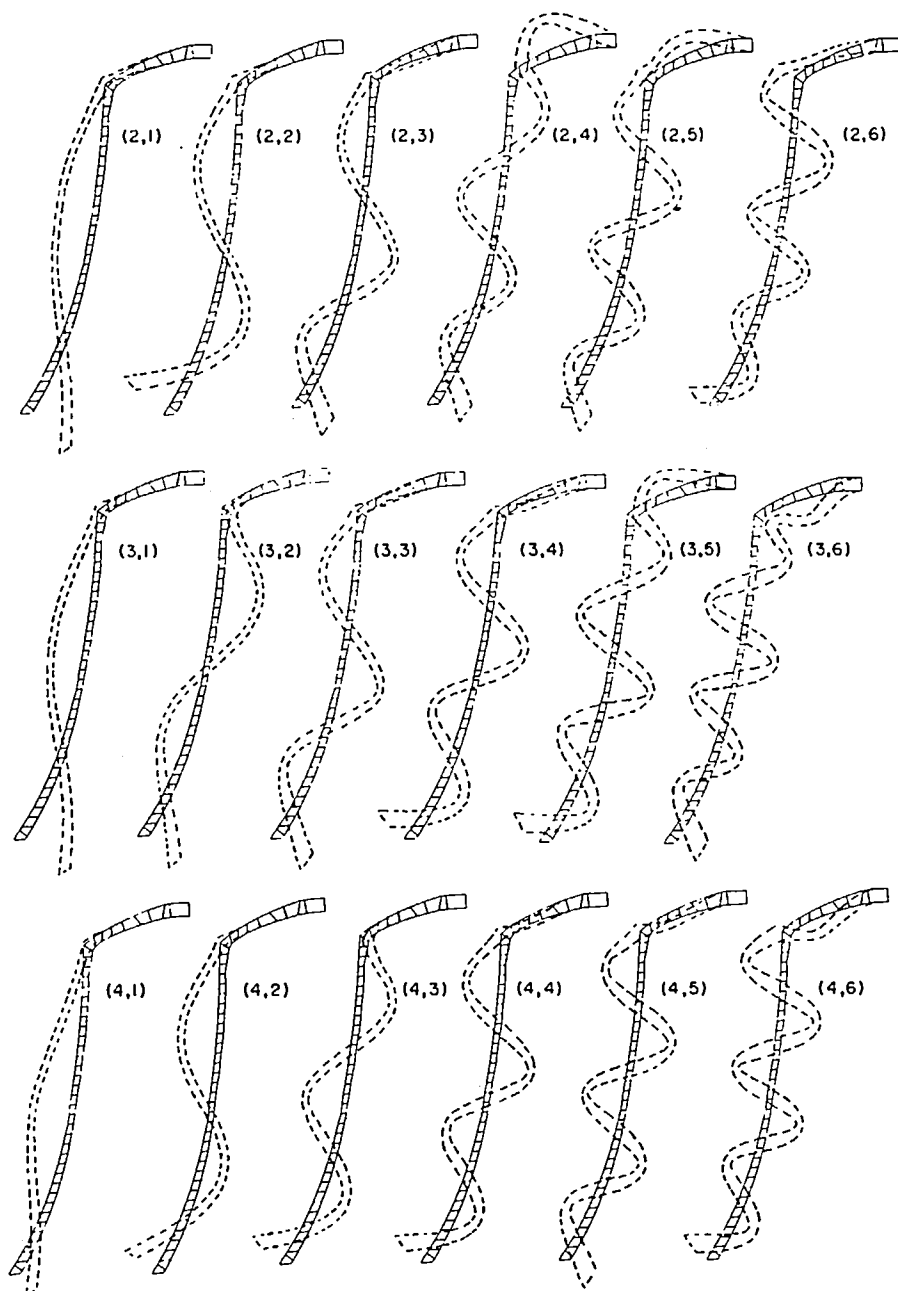


Figure 7. Some shell-driven modes for a C_3 Malmark handbell.

the church bell and to avoid the problem, one can adopt a convention of counting only circles below the shoulder: for the church bell the nodal circles in the crown have to be included.

Clearly the existence of RIR as well as shell-driven modes for the handbell shows that the rôle of the rim ring in the mode production mechanisms was overemphasized by PCD in the case of the church bell. It has therefore since been suggested [7] that "rim-driven" would be a more appropriate description than "ring-driven". If this is correct then one would expect to find a handbell equivalent of the RA mode also. This does indeed occur and the $m = 3$ case is shown in Figure 8, from which it is clearly seen to correspond to the Arnold and Warburton axial cylinder modes: it is dominated by the longitudinal w component of the bell's motion. Also its frequency is much higher than those of the RIR and $n = 1$ shell-driven modes with the same m .

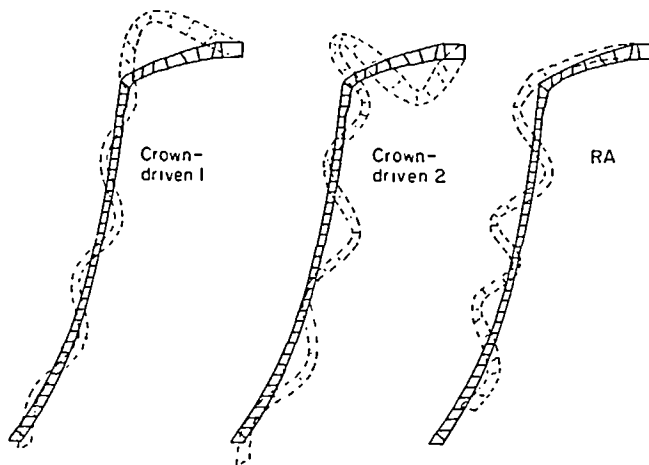


Figure 8. Crown-driven and RA modes with $m = 3$ for a C_5 Malmark handbell.

Another interesting feature from the PAFEC analysis of the handbell is the presence of "crown-driven" modes. These are not observed in the church bell, no doubt because its crown is so much thicker. They occur for each value of m and are produced essentially by the crown behaving like a flat circular plate clamped at the centre and hinged at the edges. The first two members of the family for $m = 3$ are shown in Figure 8. Such modes have not yet, to our knowledge, been observed experimentally.

4. THE SMALL CARILLON BELL

A PAFEC analysis of the rather unusual small carillon bell shown in Figure 9 reveals, as expected, the presence of RIR, shell-driven and extensional modes. Some examples are shown. Agreement with experiment is again reasonable. In this bell, like the handbell, it is the $m = 2$ and 3 RIR modes which are acoustically dominant but, like the church bell, it is the $m = 2$ case only which has no nodal circles.

An interesting feature for this bell is the appearance of what seems to be a family of "waist-driven" modes. These can presumably be attributed to the extraordinarily thin waist relative to shoulder, crown and rim for this particular bell. The first member, with $m = 2$, is shown in Figure 10. Crown participation seems to be more or less non-existent in this bell, no doubt thanks to the very massive structure in the region of the shoulder.

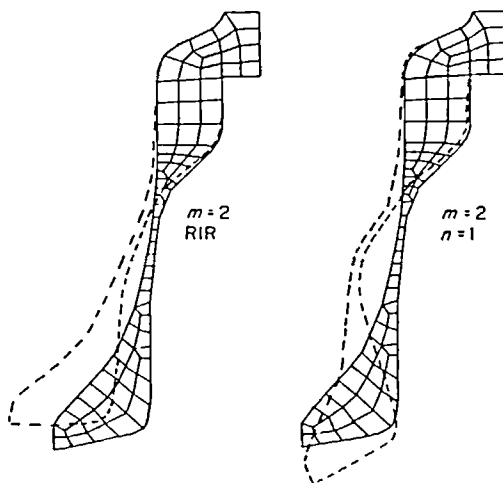


Figure 9. RIR and $n = 1$ shell-driven modes with $m = 2$ for an unusual small carillon bell.

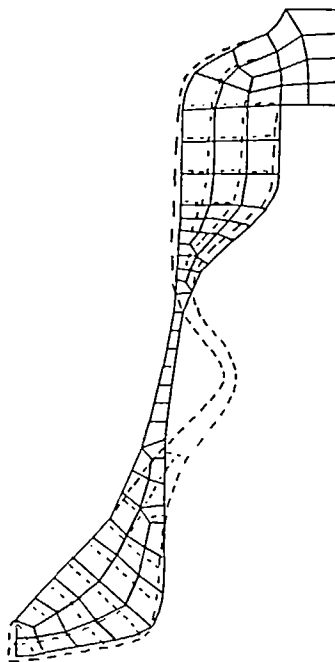


Figure 10. A waist-driven mode with $m = 2$ for a small carillon bell.

5. THE FIRE-ALARM BELL

In Figure 11 we show the first two modes with $m = 5$ for a typical modern fire-alarm bell. It is of interest mainly because, compared with other bells considered here, its crown is very wide relative to its height. The first mode shown is analogous to RIR and has no nodal circle. All m values give a very similar result, there being no change to a single nodal circle no matter how large m becomes. These RIR modes completely dominate the sound spectrum. It is because of the non-participation of the crown in these modes that bolts can be fixed through the crown and metallic labels attached without adversely influencing the sound.

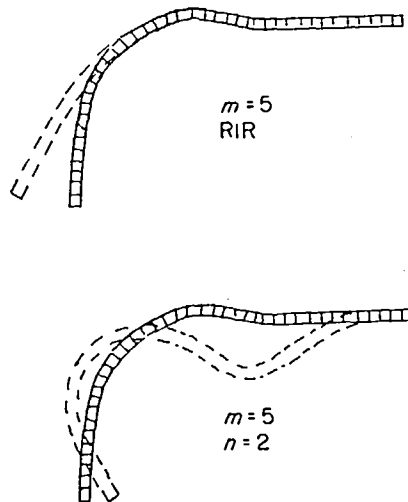


Figure 11. RIR and the first shell-driven modes with $m = 5$ for a modern alarm bell.

The second mode shown in Figure 11 is the first of the shell-driven family for $m = 5$. Crown participation has already commenced because of the large width. This has resulted in the presence of two nodal circles and is clearly related to the situation with handbells where the circle count breaks down at the point where crown participation commences. No modes with a single nodal circle ever seem to occur in these alarm bells.

6. CONCLUSIONS

A number of new conclusions can be drawn from the results we have considered here. (1) The role of the thick rim ring in church bells is less important than had previously been believed insofar as the mechanisms driving the modes are concerned. However it is still clear that it plays a major role in aligning the frequencies of the two acoustically important RIR and $n = 1$ shell-driven families. (2) The previous contention that the RIR are distinct from shell-driven modes and that the $n = 1$ modes with their circle near the rim belong to the latter family is strongly reinforced. (3) The number of RIR modes having zero nodal circles rather than one in the waist varies from the $m = 2$ only for the church and carillon bells, through the $m = 2$ and 3 for small handbells, to all m values for the fire-alarm bell. (4) After the change from zero circles to one in RIR modes of the church bell, there is no further change of regime as m increases. (5) RA modes correspond to the Arnold and Warburton axial cylinder modes and so have a very large longitudinal w component. (6) In some bells the crown take-off phenomenon is associated with a jump of 2 in the number of nodal circles as one goes up the shell-driven sequence for fixed m . (7) Crown-driven modes can occur in handbells and waist-driven modes in some small carillon bells.

ACKNOWLEDGMENTS

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